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Can conformal transformations change the fate of 2D black holes? *

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Abstract

By using a classical Liouville-type model of two dimensional dilaton gravity we show that the one-loop theory implies that the fate of a black hole depends on the conformal frame. There is one frame for which the evaporation process never stops and another one leading to a complete disappearance of the black hole. This can be seen as a consequence of the fact that thermodynamic variables are

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not conformally invariant. In the second case the evaporation always produces the same static and regular end-point geometry, irrespective of the initial state.

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The understanding of the dynamical evolution of black holes is an important ingredient in the formulation of a consistent theory of quantum gravity. Since the work of Callan-Giddings-Harvey-Strominger [1], the study of two-dimensional models for black hole formation and evaporation has increased a lot and it has been very useful to analyze quantum aspects of black hole physics. In particular, the existence of exactly solvable one-loop models [2, 3] has allowed to study back reaction effects in an analytical setting. One of the central properties of the CGHS model is that the Hawking temperature is independent of the mass and many aspects of the quantum evolution of the black holes are indeed associated with this fact. Therefore it is interesting to consider other models giving rise to black hole solutions with a more realistic Hawking temperature. In Ref [4] it was analyzed a model with a classical gravitational action conformally related to the Polyakov-Liouville action

$$S = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[R\phi + 4\lambda^2 e^{\beta\phi} - \frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right] . \qquad (1)$$

This model, closely related to the one introduced by Mann [5], has solutions which resemble the string 2D black holes [1], but the Hawking temperature depends on the mass. Moreover it also permits to construct an associated solvable semiclassical model and it was also pointed out in [4] that the one-loop solution predicts that the Hawking evaporation never stops.

The black hole solutions of the model (1) are, in an appropriate Kruskal-type gauge, of the form

$$ds^2 = \frac{-dx^+dx^-}{\frac{\lambda^2\beta}{C} + Cx^+x^-} , \qquad (2)$$

$$e^{\beta\phi} = \frac{1}{\frac{\lambda^2\beta}{C} + Cx^+x^-} , \qquad (3)$$

where the constant $|C|$ is proportional to the black hole mass. Although we cannot recover Minkowski spacetime for any value of C, there is a simple way to get a flat geometry. By performing a conformal rescaling of the metric

$$g_{\mu\nu} \to \frac{g_{\mu\nu}}{J(\phi)} \,, \tag{4}$$

where

$$J(\phi) = \frac{1}{\beta}\left(e^{\beta\phi} - 1\right) \,, \tag{5}$$

the new geometry, in conformal gauge $ds^2 = -e^{2\rho}dx^+dx^-$, is

$$ds^2 = \frac{-dx^+dx^-}{\frac{1}{\beta} - \frac{\lambda^2}{C} - \frac{C}{\beta}x^+x^-} \,, \tag{6}$$

and it is clear that for $C = \lambda^2\beta$ we obtain a Minkowskian ground state, which was not present in the original model. ¶ Moreover expanding the solutions (6) around $C = \lambda^2\beta$ we have, to leading order

$$ds^2 = \frac{-dx^+dx^-}{\beta^{-2}\lambda^{-2}(C - \beta\lambda^2) - \lambda^2 x^+x^-} \,, \tag{7}$$

which impies that, for $C \sim \beta\lambda^2$, the solutions are similar to the CGHS black hole solutions with $M = \frac{C}{\beta^2\lambda} - \frac{\lambda}{\beta}$. The above discussion serves to motivate the analysis of the conformally rescaled model through the transformation (4-5). Due to the existence of a classical Minkowski ground state one would expect that the black holes (6) may now decay completely (as, for instance, in [2] and [3]). It has already been stressed that the thermodynamical variables can depend upon the conformal frame [7] and in this paper we shall explicitly show that the evaporation process can be indeed very different.

In terms of the rescaled metric the action (1) transforms into

$$\begin{aligned} S \;\; &= \;\; \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[R\phi + \frac{\beta e^{\beta\phi}}{e^{\beta\phi}-1}(\nabla\phi)^2 \right. \\ &\left. + \frac{4\lambda^2}{\beta}e^{\beta\phi}\left(e^{\beta\phi}-1\right) - \frac{1}{2}\sum_{\imath=1}^{N}(\nabla f_i)^2\right] \,. \end{aligned} \tag{8}$$

¶This is a particular case of a general procedure to construct a theory with a flat ground state starting from an arbitrary 2D dilaton gravity theory [6].

It is also interesting to note that for $\beta = 0$ we recover the CGHS model after the trivial redefinition $\phi \to e^{-2\phi}$. Let us now consider the formation of a black hole by collapse of an infalling shock wave at $x^+ = x_0^+$. For $x^+ < x_0^+$ the metric is given by

$$ds^2 = \frac{dx^+ dx^-}{\lambda^2 x^+ x^-} \,, \tag{9}$$

with Minkowskian coordinates $\sigma^\pm$ defined by $\lambda x^\pm = \pm e^{\pm\lambda\sigma^\pm}$. After the incoming shock-wave, $x^+ > x_0^+$, the metric is described as

$$ds^2 = \frac{-dx^+ dx^-}{\frac{1}{\beta} - \frac{\lambda^2}{C} - \frac{C}{\beta}(x^+ + \Delta^+)(x^- + \Delta^-)} \,, \tag{10}$$

with

$$\Delta^+ = x_0^+ \left(\frac{\lambda^2 \beta}{C} - 1 \right) \,, \tag{11}$$

$$\Delta^- = \frac{1}{x_0^+} \left(\frac{1}{\lambda^2 \beta} - \frac{1}{C} \right) \,, \tag{12}$$

and the new asymptotically flat coordinates $\tilde{\sigma}^\pm$ are defined as $\sqrt{\frac{C}{\beta}}\,(x^\pm + \Delta^\pm) = \pm e^{\pm\sqrt{\frac{C}{\beta}}\tilde{\sigma}^\pm}$. A simple calculation leads to the following expression for the stress tensor of the shock-wave

$$T^f_{++} = \frac{C\Delta^-}{\beta} \delta(x^+ - x_0^+) \,, \tag{13}$$

and, in terms of the asymptotically flat coordinates $\sigma^\pm$, we have

$$T^f_{\sigma^+\sigma^+} = \left(\frac{C}{\lambda\beta^2} - \frac{\lambda}{\beta} \right) \delta(\sigma^+ - \sigma_0^+) \,. \tag{14}$$

This means that the energy of the wave is $M = \frac{C}{\lambda\beta^2} - \frac{\lambda}{\beta}$, which turns out to be, by energy conservation, equal to the ADM mass of the black hole solution (6). This expression is in accordance with the mass formula for generic 2D models [8, 9], with an appropriate normalization of the Killing vector k^μ at spatial infinity ($k^2 = -\frac{C}{\lambda^2\beta}$), and differs in the constant shift $-\frac{\lambda}{\beta}$ from the mass formula for the solutions (2),(3). In fact, using the arguments of [10] one can show that the mass is conformally invariant up to a constant shift (related to the choice of the ground state $\|$). We have to note that a different normalization of the Killing vector gives rise to a different expression for the mass which is not compatible with energy conservation.

The flux of radiation measured by inertial observers at future null infinity $\tilde{\sigma}^+ \to \infty$ can be worked out as follows

$$< T^f_{\tilde{\sigma}^-\tilde{\sigma}^-} >= -\frac{N}{24} \{\sigma^-, \tilde{\sigma}^-\} \ , \tag{15}$$

where $\{\sigma^-, \tilde{\sigma}^-\}$ is the Schwartzian derivative. One obtains

$$< T^f_{\tilde{\sigma}^-\tilde{\sigma}^-} >= \frac{NC}{48\beta} \left[1 - \left(1 + \Delta^- \sqrt{\frac{C}{\beta}} e^{\sqrt{\frac{C}{\beta}} \tilde{\sigma}^-} \right)^{-2} \right] \ . \tag{16}$$

At late times ($\tilde{\sigma}^- \to \infty$), the flux of radiation approaches a constant thermal value with an associated Hawking temperature given, in terms of the mass, by

$$T_H = \frac{1}{2\pi} (\lambda + \beta M) \ . \tag{17}$$

We must stress that in obtaining this expression we have used the normalization of the Killing vector already used to compute the mass and that for small masses $M << \frac{\lambda}{|\beta|}$ one recovers the constant temperature of the CGHS black hole.

We have also to point out that due to the shift Δ^+ relating the asymptotically flat coordinates σ^+ and $\tilde{\sigma}^+$ before and after the collapse $\left(e^{\lambda \sigma^+} + \lambda \Delta^+ = \lambda \sqrt{\frac{\beta}{C}} e^{\sqrt{\frac{C}{\beta}} \tilde{\sigma}^+} \right)$ there exists also an additional (semiclassical) incoming flux $< T^f_{\tilde{\sigma}^+\tilde{\sigma}^+} >$. This flux is positive and vanishes at $\tilde{\sigma}^+ \to \infty$ and one therefore could expect that it does not affect the evaporation process. Despite the

$\|$ Such a constant shift was not considered in [10] because it was supposed there that the ground state of the rescaled theory is always obtained by conformally transforming the vacuum of the original one. This is not what happens here.

presence of such a non-vanishing $< T^f_{\tilde{\sigma}^+\tilde{\sigma}^+} >$ we will be able, later, to construct evaporating solutions where this flux does not appear.

Let us now consider the semiclassical one-loop theory in both models (1) and (8). One can construct a solvable semiclassical theory, maintaining the classical free field equation $\partial_+\partial_-(2\rho - \beta\phi) = 0$, by adding a particular local counterterm to the standard non-local Polyakov effective action

$$\begin{aligned} S &= \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[R\phi + 4\lambda^2 e^{\beta\phi} - \frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right] \\ &\quad -\frac{N}{96\pi}\int d^2x\sqrt{-g}\left[R\Box^{-1}R + \beta(2R\phi - \beta(\nabla\phi)^2)\right] . \end{aligned} \tag{18}$$

We have introduced a slightly different counterterm to that of [4] because now we can also preserve the remaining unconstrained classical equation of motion (see (19)). For the theory defined by the classical action (8) a solvable one-loop theory can be obtained by transforming (18) with the rescaling (4-5). The new semiclassical theory recovers the BPP model [3] in the limit $\beta = 0$. In Kruskal coordinates ($2\rho = \beta\phi$) the equations of motion derived from (18) leads to the Liouville equation for the field 2ρ

$$\partial_+\partial_- 2\rho = -\lambda^2\beta e^{4\rho} , \tag{19}$$

and the constraint equations

$$e^{2\rho}\partial_\pm^2 e^{-2\rho} = \beta\left(T^f_{\pm\pm} - \frac{N}{12}t_\pm\right) , \tag{20}$$

where $t_\pm(x^\pm)$ are the boundary contributions coming from the non-local Polyakov term. The simplest solution can be obtained when $T^f_{\pm\pm} = 0 = t_\pm$

$$ds^2 = \frac{-dx^+dx^-}{\frac{\lambda^2\beta}{C} + Cx^+x^-} , \tag{21}$$

and, for $C < 0$ and $\lambda^2\beta < 0$, the metric (21) represents a black hole in thermal equilibrium. As it was illustrated in [4] for the CGHS case we can study the evaporation process of the black hole by considering the dynamical evolution of the solution (21) when the incoming flux is turned off at $x^+ = x_0^+$. This can be exactly achieved in the present theory by assuming the following boundary conditions

$$t_{x^+} = \frac{1}{4(x^+ + \Delta^+)^2}\Theta(x^+ - x_0^+)\,, \tag{22}$$

$$t_{x^-} = 0\,, \tag{23}$$

where

$$\Delta^+ = -x_0^+ + \frac{1}{\lambda}\left(\frac{Cx_0^+\lambda n^{\frac{1}{2}}}{C+\lambda\beta^2 M}\right)^{\frac{2}{n+1}}\,, \tag{24}$$

and

$$n^2 = 1 - \frac{N\beta}{12}\,. \tag{25}$$

The evaporating solution for $x^+ > x_0^+$ is

$$ds^2 = \frac{-dx^+dx^-}{\frac{\beta\lambda^2(\lambda x^+ + \lambda\Delta^+)^{\frac{1-n}{2}}}{n^{\frac{1}{2}}(C+\lambda\beta^2 M)} + \frac{(C+\lambda\beta^2 M)(\lambda x^+ + \lambda\Delta^+)^{\frac{n+1}{2}}(\lambda x^- + \lambda\Delta^-)}{\lambda^2 n^{\frac{1}{2}}}}\,, \tag{26}$$

where

$$\Delta^- = \frac{\lambda^2\beta}{C^2 x_0^+}\left[1 - \frac{\left(\lambda x_0^+\right)^{\frac{1-n}{1+n}}}{n^{\frac{n}{1+n}}}\left(\frac{C}{C+\lambda\beta^2 M}\right)^{\frac{2}{1+n}}\right]\,, \tag{27}$$

and there is additionally a shock wave along $x^+ = x_0^+$

$$T^f_{++} = \left[\frac{\lambda^{\frac{1-n}{1+n}}(n+1)}{2\beta}\left(\frac{C+\lambda\beta^2 M}{Cx_0^+ n^{\frac{1}{2}}}\right)^{\frac{2n}{1+n}} - \frac{1}{\beta x_0^+}\right]\delta(x^+ - x_0^+)\,. \tag{28}$$

This shock wave can be eliminated with an appropriate choice of the parameter M (M turns out to be the energy of the shock wave in the classical limit). The main property of the evaporating solution is that the apparent horizon ($\partial_+\phi = 0$)

$$\lambda x^- = -\lambda\Delta^- + \frac{\lambda^4\beta(n-1)}{(C+\lambda\beta^2 M)^2(n+1)(\lambda x^+ + \lambda\Delta^+)^n}\,, \tag{29}$$

and the singularity curve

$$\lambda^4\beta + (C+\lambda\beta^2 M)^2(\lambda x^+ + \lambda\Delta^+)^n(\lambda x^- + \lambda\Delta^-) = 0\,, \tag{30}$$

never meet each other, implying that the Hawking evaporation never ceases.

The point now is to see what happens in the evaporation process of the conformally rescaled model with $\lambda^2\beta < C < 0$, which is the range of variation of C valid for both models. The rescaled dynamical solution, after switching off the incoming flux at $x^+ > x_0^+$ is

$$\begin{aligned} ds^2 &= -\left[\frac{1}{\beta} - \frac{\lambda^2(\lambda x^+ + \lambda\Delta^+)^{\frac{1-n}{2}}}{n^{\frac{1}{2}}(C+\lambda\beta^2 M)} \right. \\ &\left. -\frac{C+\lambda\beta^2 M}{n^{\frac{1}{2}}\lambda^2\beta}(\lambda x^+ + \lambda\Delta^+)^{\frac{n+1}{2}}(\lambda x^- + \lambda\Delta^-)\right]^{-1} dx^+ dx^-\,. \end{aligned} \tag{31}$$

The curvature singularity of (31)

$$\begin{aligned} & n^{\frac{1}{2}}\lambda^2(C+\lambda\beta^2 M)(\lambda x^+ + \lambda\Delta^+)^{\frac{n-1}{2}} - \lambda^4\beta \\ & -(C+\lambda\beta^2 M)^2(\lambda x^+ + \lambda\Delta^+)^n(\lambda x^- + \lambda\Delta^-) = 0 \quad , \end{aligned} \tag{32}$$

is hidden behind the apparent horizon (29) immediately after $x^+ = x_0^+$, but they intersect each other at the point

$$x^+_{int} = -\Delta^+ + \frac{1}{\lambda}\left(\frac{2n^{\frac{1}{2}}\lambda^2\beta}{(n+1)(C+\lambda\beta^2 M)}\right)^{\frac{2}{n-1}}\quad , \tag{33}$$

$$x^-_{int} = -\Delta^- + \frac{\lambda^3\beta(n-1)}{(C+\lambda\beta^2 M)^2(n+1)}\left(\frac{2n^{\frac{1}{2}}\lambda^2\beta}{(C+\lambda\beta^2 M)(n+1)}\right)^{\frac{2n}{1-n}}\quad . \tag{34}$$

Remarkably, the evaporating solution can be matched at the end-point $x^- = x^-_{int}$ with a static stable and regular solution.

To show this let us first analyze the static radiationless solutions of the model. With the following boundary conditions

$$t_{x^\pm} = \frac{1}{4(x^\pm)^2}\,, \tag{35}$$

one can check that the solutions

$$ds^2 = \frac{-dx^+dx^-}{\frac{1}{\beta} - \frac{\lambda^2}{nC}(-\lambda^2 x^+x^-)^{\frac{1-n}{2}} + \frac{C}{n\lambda^2\beta}(-\lambda^2 x^+x^-)^{\frac{n+1}{2}}}\,, \tag{36}$$

where C is an integration constant and n is given by (25), are stable with respect to the asymptotically Rindlerian coordinates $\sigma^\pm$ defined by

$$\lambda x^\pm = \pm e^{\pm\frac{C}{\lambda\beta}\sigma^\pm}\,. \tag{37}$$

The requirement of stability with respect to these coordinates seems natural because in terms of them the solutions (36) are independent of the time coordinate (similar considerations using asymptotically Rindler coordinates in solvable models of 2D gravity can also be found in [11]). For $C < \hat{C}$, where

$$\hat{C} = -\lambda^2(-\beta)^{\frac{n+1}{2}}\frac{2^n}{(n+1)^{\frac{n+1}{2}}(n-1)^{\frac{n-1}{2}}}\,, \tag{38}$$

the spacetime geometry has a naked singularity, for $C = \hat{C}$ the solution is completely regular and for $C > \hat{C}$ there are null singularities at $x^+x^- = 0$. Alternatively one can also choose these solutions to be stable with respect to the asymptotically Minkowskian frame although in these coordinates the solutions (36) are no longer static (the same prescription was adopted in [12]) **. In this case the boundary functions (35) must be substituted by

$$t_{x^\pm} = \frac{2n - n^2 + 3}{16(x^\pm)^2}\,, \tag{39}$$

and n is now given by

$$n = \frac{2 - \frac{N\beta}{8}}{2 - \frac{N\beta}{24}}\,. \tag{40}$$

Analogously, in order to eliminate the incoming flux in the evaporating solution (26) the boundary function (22) would be substituted by

$$t_{x^+} = \frac{2n - n^2 + 3}{16(x^+ + \Delta^+)^2}\Theta(x^+ - x_0^+)\,. \tag{41}$$

**We must point out that in the classical limit both descriptions coincide.

As we have already mentioned the evaporating solution (31) can be just matched with the static and regular solution with $C = \hat{C}$

$$ds^2 = -\left[\frac{1}{\beta} - \frac{\lambda^2}{n\hat{C}}(\lambda x^+ + \lambda\Delta^+)^{\frac{1-n}{2}}(-\lambda x^- - \lambda\tilde{\Delta}^-)^{\frac{1-n}{2}} + \frac{\hat{C}}{n\lambda^2\beta}(\lambda x^+ + \lambda\Delta^+)^{\frac{n+1}{2}}(-\lambda x^- - \lambda\tilde{\Delta}^-)^{\frac{n+1}{2}}\right]^{-1} dx^+ dx^- \ , \quad (42)$$

where

$$\tilde{\Delta}^- = n\Delta^- + (n-1)x^-_{int} \ , \quad (43)$$

and there is emission of a"thunderpop" along the null line $x^- = x^-_{int}$

$$T^f_{--} = \frac{1-n}{2n\beta(x^-_{int} + \Delta^-)}\delta(x^- - x^-_{int}) \ . \quad (44)$$

We can obtain the same result analizing the evaporation process of a black hole formed by gravitational collapse. The static radiationless solutions (36) $(C \leq \hat{C})$ can be matched at the shock wave line $x^+ = x^+_0$ with an evaporating solution

$$ds^2 = \frac{-dx^+dx^-}{\frac{1}{\beta} - \frac{\lambda^2((\lambda x^+ + \lambda\Delta^+)(-\lambda x^-))^{\frac{1-n}{2}}}{n(C+\lambda\beta^2 M)} + \frac{C+\lambda\beta^2 M}{n\lambda^2\beta}\frac{(\lambda x^+ + \lambda\Delta^+)^{\frac{n+1}{2}}((-\lambda x^-)^n - \lambda\Delta^-)}{(-\lambda x^-)^{\frac{n-1}{2}}}} \ , \quad (45)$$

with

$$\Delta^+ = x^+_0\left[\left(\frac{C}{C+\lambda\beta^2 M}\right)^{\frac{2}{1+n}} - 1\right] \ , \quad (46)$$

$$\Delta^- = \frac{\lambda^3\beta}{(\lambda x^+_0)^n C^2}\left[1 - \left(\frac{C}{C+\lambda\beta^2 M}\right)^{\frac{2}{1+n}}\right] \ , \quad (47)$$

and the energy momentum tensor takes the expression

$$T^f_{++} = \frac{n+1}{2\beta x^+_0}\left[\left(\frac{C+\lambda\beta^2 M}{C}\right)^{\frac{2}{n+1}} - 1\right]\delta(x^+ - x^+_0) \ . \quad (48)$$

The singularity curve of (45)

$$n\lambda^2(C+\lambda\beta^2 M)((\lambda x^+ + \lambda\Delta^+)(-\lambda x^-))^{\frac{n-1}{2}} - \lambda^4\beta + (C+\lambda\beta^2 M)^2(\lambda x^+ + \lambda\Delta^+)^n((-\lambda x^-)^n - \lambda\Delta^-) = 0 \quad , \quad (49)$$

is hidden behind the apparent horizon

$$(-\lambda x^-)^n = \lambda\Delta^- + \frac{\lambda^4\beta(1-n)}{(C+\lambda\beta^2 M)^2(1+n)(\lambda x^+ + \lambda\Delta^+)^n}\,, \tag{50}$$

provided the mass M is above a critical mass M_{cr} (vanishing for $C = \hat{C}$), and the black hole shrinks until the intersection point

$$\begin{aligned} x^+_{int} &= -\Delta^+ + \lambda^{-\frac{1+n}{n}}(\Delta^-)^{-\frac{1}{n}}\left[\left(\frac{2\lambda^2\beta}{(C+\lambda\beta^2 M)(1+n)}\right)^{\frac{2n}{n-1}}\right. \\ &\quad \left. - \frac{\lambda^4(1-n)}{(C+\lambda\beta^2 M)^2(1+n)}\right]^{\frac{1}{n}}\,, \end{aligned} \tag{51}$$

$$\begin{aligned} x^-_{int} &= -\frac{(\lambda\Delta^-)^{\frac{1}{n}}}{\lambda}\left(\frac{\lambda^2\beta}{(C+\lambda\beta^2 M)(1+n)}\right)^{\frac{2}{n-1}}\left[\left(\frac{2\lambda^2\beta}{(C+\lambda\beta^2 M)(1+n)}\right)^{\frac{2n}{n-1}}\right. \\ &\quad \left. - \frac{\lambda^4(1-n)}{(C+\lambda\beta^2 M)^2(1+n)}\right]^{-\frac{1}{n}}\,. \end{aligned} \tag{52}$$

The evaporating solution can also be matched across $x^- = x^-_{int}$ with the regular static solution (42) where now

$$\tilde{\Delta}^- = \Delta^-(-\lambda x^-_{int})^{1-n}\,, \tag{53}$$

and with emission of a "thunderpop" at $x^- = x^-_{int}$

$$T^f_{--} = \frac{1-n}{2\beta x^-_{int}}\left[\left(\frac{C+\lambda\beta^2 M}{\hat{C}}\right)^{\frac{2}{1-n}} - 1\right]\delta(x^- - x^-_{int})\,. \tag{54}$$

We want to remark that the different result for the process of black hole evaporation in the initial Liouville-type model (1) and in the rescaled one (8) can be seen as a consequence of the different relation between thermodynamic variables in each of these models. In the model (1) the temperature is proportional to the black hole mass and therefore goes to zero when the black hole mass becomes very small preventing the complete evaporation. On the contrary in the model (8) due to the shift in the temperature (17) it approaches a constant non-vanishing value $\frac{\lambda}{2\pi}$ when the black hole mass goes to zero and consequently the black hole evaporates completely. In this second model the evaporation always ends with the same remnant geometry, irrespective of the initial state or the type of evaporation process (thermal bath removal or gravitational collapse). We find that this end-point geometry is everywhere regular, as in other models [2, 3] for which the evaporation process can be followed analytically, suggesting an underlying general behaviour. Moreover the exact solvability of these models can be used to analyze other physical aspects in an analytical setting (critical behaviour, thermality, etc). We will consider these questions and details of this work in a future publication.

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